Localization of Acoustic Sources through the Fitting of Propagation Cones.

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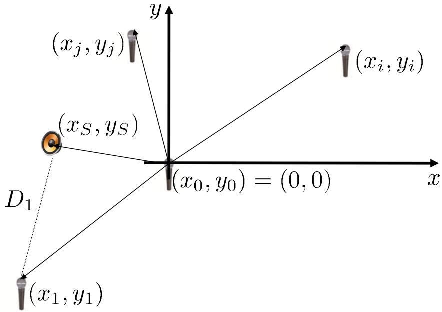
2D Source Localization

Problem: point-like acoustic source localization based on time differences of arrival (TDOA) measurements between distinct microphones pairs.



- Source coordinates: (x_S, y_S). Time signal emission: t_S.
- Microphones coordinates: (x_i, y_i), $i = 0, \dots, N$.
  Time of arrival (TOA) of the signal at microphone i: t_i.
- Reference microphone: $(x_0, y_0) := (0, 0)$, $t_0 := 0$.
- TDOA between microphones i and j: $\tau_{ij} = t_i - t_j$.

Motivation and Goals

Acoustic source localization is an important problem in many diverse applications such as:

- seismic remote sensing;
- military surveillance and reconnaissance;
- environmental and wildlife habitat monitoring.

Open problems in modern working scenario:

- growing interest in situations with **multiple unsynchronized arrays of microphones**, as integrated digital arrays of microphones become accessible at low cost.
- microphone clusters deployment are subject to constraints of different nature (e.g. architectural), therefore it is necessary to be able to accurately and swiftly **predict the localization accuracy** for a given geometry of clusters.

State of the art

The Maximum Likelihood Estimators: [Hahn 73; Wax 83; Stoica 88; Chen 02; Huang 04; Castaneda 06]

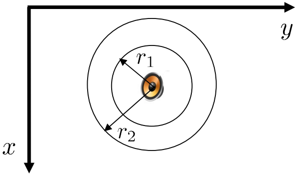
- well-known advantage of asymptotic efficiency for a wide sample space;
- they need statistical characterization of the measurements.

Least Squares Estimators, based on minimization of a squared error function incorporating both source location and measurements. Some relevant examples are based on:

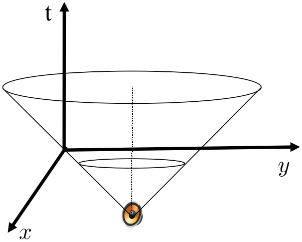
- intersection of planes specified by different sensor triplets [Schmidt 87];
- spherical intersection (closed-form) [Schau 87];
- intersection between hyperbolas associated to the TDOAs of different pairs of sensors [Chan 94];
- hyperbolic error function defined as the difference between the observed TDOA and the one generated by a signal model depending on the unknown parameters [Huang 01].
- a closed-form solution for the LS error criterion that partly addresses the case of multiple arrays [Gillette 08].

2D Propagation Model

- In a homogeneous and isotropic medium, an acoustic wavefront that propagates from a point-like source is a **circle centered in the source location, whose radius grows linearly with time.**

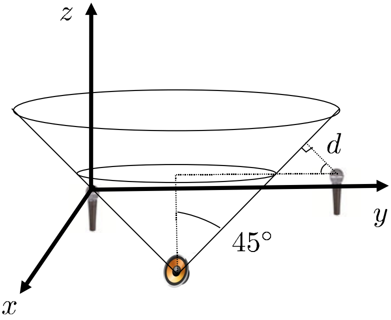


- In 3D spacetime wavefront propagation defines a cone surface, the **propagation cone**.
- The source spacetime position coincides with the **cone apex**.



2D Propagation Model

- Reference microphone m_0 with spacetime coordinates $(x_0, y_0, z_0) = (0, 0, 0)$.
- Microphones m_i with spacetime coordinates $(x_i, y_i, z_i = c \cdot \tau_{i0})$, $i = 1, \ldots N$, where c is the speed of sound.
- Source S with spacetime coordinates (x_S, y_S, z_S).



Cone equation: $(x - x_S)^2 + (y - y_S)^2 - (z - z_S)^2 = 0.$

Cone distance: $d = \frac{\sqrt{2}}{2} \left| \sqrt{(x - x_S)^2 + (y - y_S)^2} - (z - z_S) \right|.$

Cone Localization

The Key Idea:

- in an error free scenario each measurement defines a spacetime point lying on the cone surface;
- with noisy measurements **we have to search for the best fit cone**.

Cost functions: let us define two least square cost functions

$$J_e(x_S, y_S, z_S) := \sum_{i=0}^{N} \epsilon_{e,i}^2 \qquad J_d(x_S, y_S, z_S) := \sum_{i=0}^{N} \epsilon_{d,i}^2,$$

where $\epsilon_{e/d,i}$ is an estimate for the error on the i measurement:

- equation error: $\epsilon_{e,i} := (x_i - x_S)^2 + (y_i - y_S)^2 - (z_i - z_S)^2$;
- distance error: $\epsilon_{d,i} := \sqrt{(x_i - x_S)^2 + (y_i - y_S)^2} - (z_i - z_S)$.

Localization: $$(\hat{x}_s, \hat{y}_s) = \underset{(x,y)}{\text{argmin}}\, J_{e/d}(x_S, y_S, z_S).$$

Single Array Simulations: Bias

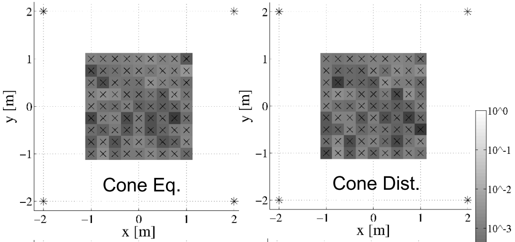
- bias abs. value on x:

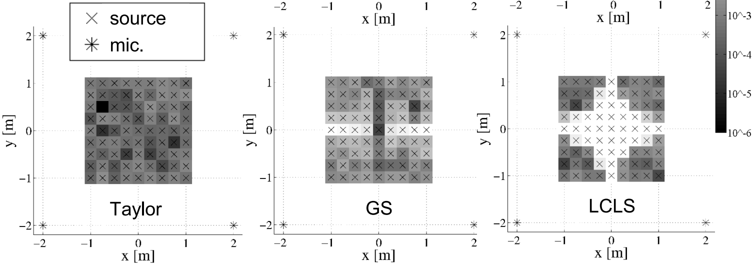
$$b_x = \left| \frac{1}{n} \sum_{i=1}^{n} (x_s - \hat{x}_{s,i}) \right|$$

n=50 simulations

- additive noise:

$$\sigma_{noise} = 2cm$$



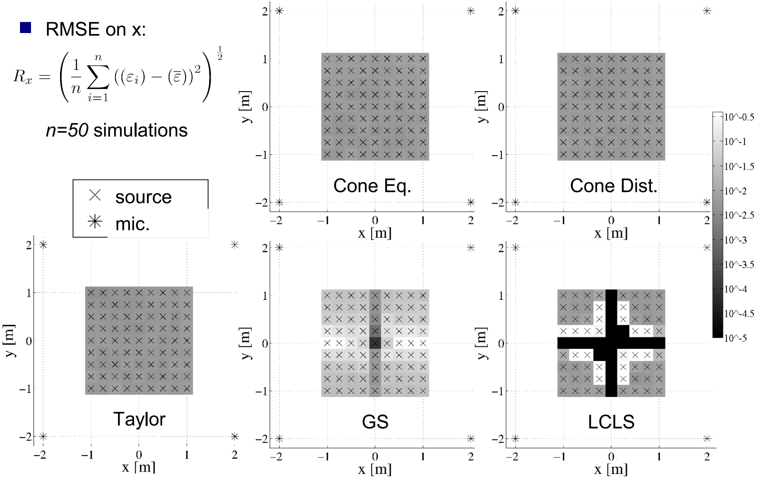


Single Array Simulations: RMSE

- RMSE on x:

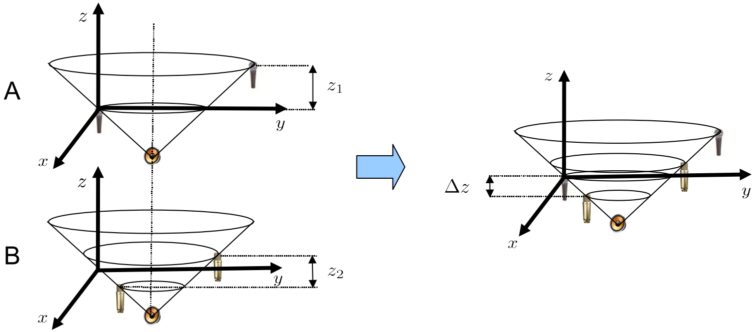
$$R_x = \left(\frac{1}{n} \sum_{i=1}^{n} \left((\varepsilon_i) - (\bar{\varepsilon}) \right)^2 \right)^{\frac{1}{2}}$$

n=50 simulations



Propagation Cone Without a Global Synchronization

If microphones arrays are not globally synchronized we have **a cone for each cluster of microphones, each traslated on the distance propagation axis.**



To avoid the introduction of new variables, we have to estimate the new parameters Δz.

Multiple Arrays Cone Localization

The Key Idea: We estimate $\Delta z^{(l)}$ with the error free model measurement:

$$\Delta z^{(l)} := c \cdot (\tau_0^{(l)} - \tau_0^{(1)})$$
$$= \sqrt{(x_0^{(l)} - x_S)^2 + (y_0^{(l)} - y_S)^2} - \sqrt{(x_0^{(1)} - x_S)^2 + (y_0^{(1)} - y_S)^2}.$$

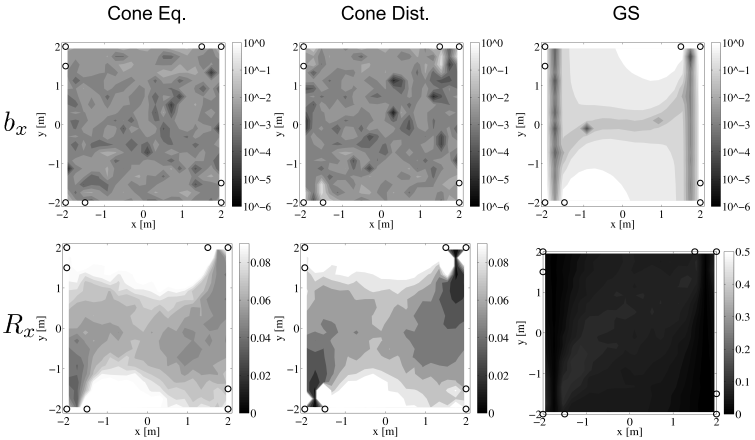
Cost functions: we modify the above cost functions

$$J_e(x_S, y_S, z_S) := \sum_{i=0}^{N} {\epsilon_{e,i}^{(l)}}^2 \qquad J_d(x_S, y_S, z_S) := \sum_{i=0}^{N} {\epsilon_{d,i}^{(l)}}^2,$$

- $\epsilon_{e,i}^{(l)} := (x_i^{(l)} - x_S)^2 + (y_i^{(l)} - y_S)^2 - (z_i^{(l)} + \Delta z^{(l)} - z_S)^2;$
- $\epsilon_{d,i}^{(l)} := \sqrt{(x_i^{(l)} - x_S)^2 + (y_i^{(l)} - y_S)^2} - (z_i^{(l)} + \Delta z^{(l)} - z_S);$

Localization: $(\hat{x}_s, \hat{y}_s) = \underset{(x,y)}{\operatorname{argmin}}\ J(x_S, y_S, z_S).$

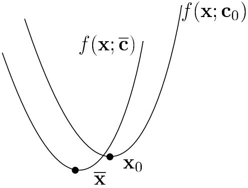
Multiple Arrays Simulations



Error Analysis

Generic cost function $f(\mathbf{x};\mathbf{c})$:

- variables $\mathbf{x}$ are the objects of the estimation (source position);
- parameters $\mathbf{c}$ are the experimental measurements (TDOA).



Problem:

- free error measurements $\mathbf{c_0}$: the minimum $\mathbf{x_0}$ of $f(\mathbf{x};\mathbf{c_0})$ coincides with the actual source position;
- noisy parameters $\bar{\mathbf{c}} = \mathbf{c_0} + \delta\mathbf{c}$: it occours a displacement in the minimum position $\bar{\mathbf{x}} = \mathbf{x_0} + \delta\mathbf{x}$.

Goal: to obtain an analytic relation between $\delta\mathbf{x}$ and $\delta\mathbf{c}$.

Error Analysis

Solution:

- The **linear response** $\delta\mathbf{x}$ with respect to $\delta\mathbf{c}$ is:

$$\mathbf{A} := -\mathbf{H}_{\mathbf{x},\mathbf{x}}(\mathbf{f})|^{-1}_{\mathbf{x_0},\mathbf{c_0}} \cdot \mathbf{H}_{\mathbf{c},\mathbf{x}}(\mathbf{f})|_{\mathbf{x_0},\mathbf{c_0}} \quad \Rightarrow \quad \delta\mathbf{x} = \mathbf{A}\ \delta\mathbf{c}.$$

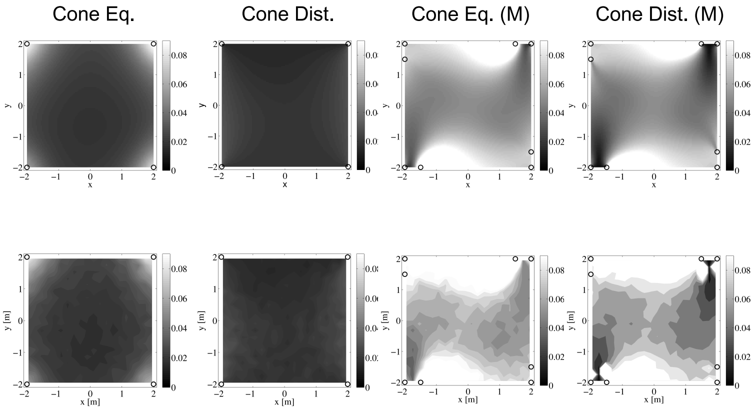
- The covariance matrix $\mathbf{M}_x$ of the source position and the covariance matrix $\mathbf{M}_c$ of the measurements are related by

$$\mathbf{M}_x = \mathbf{A}^T\mathbf{M}_c\mathbf{A}.$$

Limits: the linear relation

- is valid only under the assumptions of small errors and small bias;
- is not able to estimate bias.

Error Analysis Validation



Conclusions and Perspective

Conclusions:

- ability to exploit measurements from many sensors/arrays;
- single array: comparable to the state of the art;
- multiple arrays: good behavior respect to the state of the art;
- theoretical error analysis.

Perspective:

- extension to 3D localization;
- extension to multi-source localization in reverberant environments;
- theoretical error analysis in general case with large measurement errors and accounting for bias.