POLITECNICO DI MILANO

PhD Course
Politecnico di Milano
November 14 – 18, 2016

Course Title
Discrete and Geometric Tomography

Professor Responsible
Paolo Dulio

Contacts at campus Leonardo Milano

Tel
+39 02 2399 4577
Email:
paolo.dulio@polimi.it

Politecnico di Milano
Dipartimento di Matematica
"F. Brioschi"
Via Bonardi 9
20133 Milano

The course will take place at the Department of Mathematics, via Bonardi, Milano (Building *NAVE*).
On **Monday, November 14** we will meet at the ground floor at 9.00.
Lectures are at the 3th and the 4th floor

Overview

COMPUTERIZED AXIAL TOMOGRAPHY (CT)
General principles.
The Radon Transform.
Main theoretical reconstruction model.
Examples and applications.

DISCRETE TOMOGRAPHY (DT)
Discrete models
Ghosts and switching components.
Ryser algorithm.
Algebraic approach.
Uniqueness models.
Uniqueness and additivity.
Some reconstruction algorithms
Examples and applications.

GEOMETRIC TOMOGRAPHY (GT)
Hammer's problem.
Parallel and point X-rays
Tomography for special geometric objects.
Examples and applications

DISCUSSIONS
Open problems
Research projects

MAIN REFERENCES

1) Richard Gardner, ***Geometric Tomography***, *Cambridge University Press, New York*, second edition, 2006.
2) Gabor T. Herman and Attila Kuba Eds***., Advances in discrete tomography and its applications***, Applied and Numerical Harmonic Analysis. *Birkhäuser Boston, Inc., Boston, MA,* 2007.

Further references will be given during the course, and cited papers will be supplied to all interested students

REFERENCE PAPERS

[1] S. van Aert, K.J. Batenburg, M.D. Rossell, R. Erni, and G. vanTendeloo, *Three-dimensional atomic imaging of crystalline nanoparticles*, Nature, 470, (2011), pp. 374-377.

[2] A. Alpers and S. Brunetti, *Stability results for the reconstruction of binary pictures from two projections*, Image and Vision Computing 25, (2007), pp. 1599–608.

[3] A. Alpers, P. Gritzmann, and L. Thorens, *Stability and instability in discrete tomography* , in Digital and Image Geometry 2000, Lecture Notes in Computer Science 2243, Springer. Berlin, 2001, pp. 175–86.

[4] E. Barcucci, A. Del Lungo, M. Nivat and R. Pinzani, X-ray characterizing some classes of discrete sets, *Linear Algebra Appl.* 339 (2001), 3-21.

[5] J. Batenburg, W. Fortes, L. Hajdu, R. Tijdeman**,** *Bounds on the quality of reconstructed images in binary tomography*, Discrete Applied Mathematics, 161 (15), (2013), 2236-2251

[6] H.S.M. Coxeter, Affinely regular polygons, *Abh. Math. Sem. Univ. Hamburg* **34** (1970), 38-58.

[7] H.S.M. Coxeter, Affine regularity, *Abh. Math. Sem. Univ. Hamburg* **62** (1992), 249-53.

[8] S. Brunetti and A. Daurat, *An algorithm reconstructing lattice convex sets*, Theoret. Comp. Sci. 304 (2003), pp. 35–57.

[9] S. Brunetti and A. Daurat, *Stability in discrete tomography: Some positive results*, Discrete Appl. Math 147 (2005), pp. 207–26.

[10] Brunetti S., Dulio P., Peri C., *Discrete tomography determination of bounded lattice sets from four X-rays*, Discrete Applied Mathematics, 161 (15) (2013), 2281-2292.

[11] Brunetti S., Dulio P., Hajdu L., Peri C., *Ghosts in Discrete Tomography*, Journal of Mathematical Imaging and Vision, 53, (2015) 210-224

[12] Brunetti S., Dulio P., Peri C., Discrete tomography determination of bounded sets in Z^n, Discrete Applied Mathematics, 183 (11), (2015), 20–30

[13] B. E. van Dalen, *Stability results for two directions in discrete tomography*, Discrete Math. 309 (2009), pp. 3905–16.

[14] B. E. van Dalen, *On the difference between solutions of discrete tomography problems*, Journal of Combinatorics and Number Theory 1 (2009), pp. 15–29.

[15] B. E. van Dalen, *On the difference between solutions of discrete tomography problems II*, Pure Mathematics and Applications, 20 (2009) N.1-2 pp. 103–12.

[16] Dulio P., Frosini A., Pagani S., *A geometrical characterization of regions of uniqueness and applications to discrete tomography*, Inverse Problems, 31(12), (2015), pp. 125011

[17] Dulio P.- Peri C., *Discrete Tomography and Plane Partitions*, Advances in Applied Mathematics, 50 (3), (2013), 390-408.

[18] Dulio P.- Peri C., *Discrete tomography for inscribable lattice sets*, Discrete Applied Mathematics,161 (13-14) (2013), 1959-1974.

[19] Dulio P. *Convex decomposition of U-polygons*, Theoretical Computer Science, 406/1-2, (2008), 80-89

[20] Dulio P.- Peri C., *On the geometric structure of lattice U-polygons*, Discrete Math., 307/19-20 (2007), 2330-2340

[21] Dulio P.- Gardner R.J.- Peri C., *Discrete point X-rays*, SIAM J. Discrete Math. 20, no. 1 (2006), 171-188.
[22] P. C. Fishburn, J. C. Lagarias, J. A. Reeds, and L. A. Shepp, *Sets uniquely determined by projections on axes I. Continuous case*, SIAM J. Appl. Math. 50 (1990), pp. 288-306.
[23] P. C. Fishburn, J. C. Lagarias, J. A. Reeds, and L. A. Shepp, *Sets uniquely determined by projections on axes II. Discrete case*, Discrete Math. 91 (1991), pp.149-159.
[24] P. C. Fishburn and L. A. Shepp, *Sets of uniqueness and additivity in integer lattices*, in: Discrete Tomography: Foundations, Algorithms and Application, ed. by G. T. Herman and A. Kuba, Birkh¨auser, Boston, 1999, pp. 35–58.
[25] J.C. Fisher and R.E. Jamison, Properties of affinely regular polygons, *Geom. Dedicata* **69** (1998), 241-59.
[26] R. J. Gardner, *Sets determined by finitely many X-rays*, Geom. Dedicata 43 (1992), pp. 1–6.
[27] R. J. Gardner, *Geometric Tomography*, 2nd ed. Cambridge University Press, New York, 2006.
[28] R. J. Gardner and P. Gritzmann, *Discrete tomography: Determination of finite sets by X-rays*, Trans. Amer. Math. Soc. 349 (1997), pp. 2271–2295.
[29] R. J. Gardner and P. Gritzmann, *Uniqueness and complexity in discrete tomography*, in: Discrete Tomography: Foundations, Algorithms and Application, ed. by G. T. Herman and A. Kuba, Birkh¨auser, Boston, 1999, pp. 85–113.
[30] R. J. Gardner, P. Gritzmann and P. Prangenberg, *On the reconstruction of binary images from their discrete Radon transform*, in: Vision Geometry V, ed. by R. A. Melter, A. Y.Wu, and L. Latecki, Society of Photo-Optical Instrumentation Engineers Proceedings 2826, 1996, pp. 121–132.
[31] R.J. Gardner and P. McMullen, On Hammer's *X*-ray problem, *J. London Math. Soc.* (2) **21** (1980), 171-175.
[32] O. Giering, Bestimmung von Eibereichen und Eik¨orpen durch Steiner-Symmetrisierungen, Sber. Bayer. Akad. Wiss. M¨*u*nchen, Math.-Nat.Kl. (1962) 225-253.
[33] J. Hadamard, *Lectures on Cauchy's Problem in Linear Partial Differential Equations*, Yale University Press, New Haven, CT, 1923.
[34] L. Hajdu and R. Tijdeman, Algebraic aspects of discrete tomography, *J. reine angew. Math* **534** (2001), 119-128.
[35] P. C. Hammer, Problem 2, in: V.L. Klee (Ed.), Proc. Symp. in Pure Mathematics, vol. VII: Convexity, American Mathematical Society, Providence, RI, 1963, pp. 498-499.
[36] G. T. Herman and A. Kuba, *Discrete Tomography: Foundations, Algorithms, and Applications*, Birkh¨auser, Boston, 1999.
[37] G. T. Herman and A. Kuba, *Advances in Discrete Tomography and its Applications*, Birkh¨auser, Boston, 2007.
[38] C. Kisieloski, P. Schwander, F. H. Baumann, M. Seibt, Y. Kim, and A. Ourmazd, *An approach to quantitative high-resolution transmission electron microscopy of crystalline materials*, Ultramicroscopy 58 (1995), pp. 131–55.
[39] R. W Irving and M. R. Jerrum, *Three-dimensional statistical data security problem*, SIAM J. Comput. 23 (1994), pp. 170–84.

[40] J.R. Jinschek, K.J. Batenburg, H. Calderon, D. Van Dyck, F.R. Chen, C. Kisielowski, *Prospects for bright field and dark field electron tomography on a discrete grid*, Microscopy Microanal. 10 (Suppl. 3) (2004), pp. 44–45. Cambridge Journals Online.
[41] G.G. Lorentz, A problem of plane measure, *Amer. J. Math.* **71** (1949), 417-426.
[42] P. A. MacMahon *Combinatory Analysis*, Vols. I,II. Cambridge Univ. Press. Cambridge, England, 1916. (Reprinted by Chelsea, New York, 1960).
[43] F. Natterer, The Mathematics of Computerized Tomography, Teubner, Stuttgart, 1986.
[44] G. Ólafsson, E. Todd Quinto (Eds.), The Radon Transform, Inverse Problems, and Tomography, American Mathematical Society, Boston, MA, USA, 2006.
[45] J. Radon ¨*Uber die Bestimmung von Funktionen durch ihre Integralwerte langs gewisser Mannigfaltigkeiten*, Ber. Verh. Sachs. Akad. Wiss. Leipzig Math.-Phys. Kl. **69** (1917), 262-277
[46] E. Vallejo *Reduction of additive sets, sets of uniqueness and pyramids*, Discrete Math. 173 (1997) 257-267.
[47] E. Vallejo *A characterization of additive sets*, Discrete Math. 259 (2002) 201-210.
[48] A. Volcic, *Well-posedness of the Gardner-McMullen reconstruction problem*, Proc. Conf. Measure Theory, Oberwolfach 1983, Lecture Notes in Mathematics 1089, Springer, Berlin, 1984, pp. 199–210.

Further references will be given during the course, and a few cited papers will be supplied.

PROGRAM

Monday, 14 Morning 9:15-13:15
Overview of the Course. A brief history of CAT. Qualitative description of the Radon transform and its inversion for X-ray image reconstruction. Applications and related problems. The origin of Geometric Tomography and of Discrete Tomography. Continuous and discrete parallel X-rays. Continuous and discrete point X-rays. Remarks and examples.
Afternoon 2:30-4:30 Discussion and exercise section.

Tuesday, 15 Morning 9:15-13:15
Projections of lattice sets with discrete parallel X-rays. The reconstruction problem in Discrete Tomography. Linear Systems and Singular Value Decomposition. Ryser algorithm. Bad configurations, weakly bad configurations, switching components, ghosts. Ridge functions and additivity.
Afternoon 2:30-4:30 Discussion and exercise section.

Wednesday, 16 Morning 9:15-13:15
Algebraic approach in a finite grid and characterization of switching components. Uniqueness models in discrete tomography. Uniqueness and additivity. Reconstruction with suitable sets of four directions. Characterization of region of interests in a finite lattice grid. Remarks on applications and examples.
Afternoon 2:30-4:30 Discussion and exercise section.

Thursday, 17 Morning 9:15-13:15
Geometric Tomography, Hammer's problem and related uniqueness problems. Mid-point construction. U-polygons and their properties. The theorem of Gardner-McMullen in the Euclidean plane. The results of Gardner and Gritzmann in the integer lattice. Projections of convex bodies with point X-rays. The theorem of Volcic in the Euclidean plane. P-polygons. Some results and examples in the lattice.
Afternoon 2:30-4:30 Discussion and exercise section.

Friday, 18 Morning 9:15-12:15 *Survey on the presented results, deepening, comments and supplementary material.*
Afternoon 2:30-4:30 Discussion, open problems and possible research projects

PROGRAM OF THE COURSE

Overview of the Course. A brief history of CAT. The Radon transform and its inversion for X-ray image reconstruction. Applications and related problems. Discretization of the reconstruction process. The Matlab"radon" and "iradon" functions. Examples and applications. Reconstruction from a limited number of projections. The problem of ghosts.

The origin of Geometric Tomography and of Discrete Tomography. Continuous and discrete parallel X-rays. Continuous and discrete point X-rays. Remarks and examples.

Projections of lattice sets with discrete parallel X-rays. Description of the main models for discrete tomography. Algebraic approach. The reconstruction problem in the grid model as a linear system of equations. Some remarks on Singular Value Decomposition and on stability of solutions.

Binary Tomography. Bad configurations, weakly bad configurations, switching components, ghosts. Ryser algorithm and a few extensions. Examples of binary reconstruction and characterization of the solutions. Ridge functions and additivity.

Algebraic approach in a finite lattice grid and polynomial characterization of switching components. Uniqueness models in discrete tomography. Uniqueness and additivity. Reconstruction with suitable sets of four directions. Characterization of regions of interest in a finite lattice grid. Remarks on possible applications and examples.

Geometric Tomography, Hammer's problem and related uniqueness problems. Mid-point construction. U-polygons and their properties. The theorem of Gardner-McMullen in the Euclidean plane. The results of Gardner and Gritzmann in the integer lattice. Projections of convex bodies with point X-rays. The theorem of Volcic in the Euclidean plane. P-polygons. Some results and examples in the lattice.

EXPECTED RESULTS

The aim of the course is to provide an introduction to Discrete and Geometric Tomography, and to some related research problems. Moving from Computerized Axial Tomography, the focus is naturally turned on the discretization process. Students are expected to learn the main theorems, and the usual approaches to the reconstruction problem from a finite number of projection.

For evaluation, students can choose one of the following options

- Reading one of the research papers cited during the course, and reporting on the corresponding results. Answering possible related questions from the teacher.
- Writing a Matlab code concerning a discussed reconstruction problem. Running the code on different phantom images and producing the corresponding reconstructions.
- Presenting and discussing a possible research project concerning Discrete Tomography. The project should be based on some preliminary result or conjecture.
- Answering questions concerning the topics treated during the course. Detailed proofs of the presented theorems are required