PAOLO LELLA

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Education

- *PhD in Mathematics*, Università degli Studi di Torino, February 2012. Thesis title: "Computable Hilbert schemes". Advisor: Prof. Margherita Roggero.
- *Master degree in Mathematics*, 110/110 summa cum laude, Università degli Studi di Torino, July 2008. Thesis title: "Hilbert schemes". Advisor: Prof. Margherita Roggero.
- *Bachelor degree in Mathematics*, 110/110 summa cum laude, Università degli Studi di Torino, July 2006. Thesis title: "Factorization methods: an algorithmic approach". Advisor: Prof. Umberto Cerruti.

Employment

Current

- From January 2021. *Associate Professor*, Mathematics Department, Politecnico di Milano.

Past appointments

- January 2018–January 2021. *Assistant Professor (Ricercatore RTDb)*, Mathematics Department, Politecnico di Milano.
- September 2017–January 2018. *Assistant Professor (Ricercatore RTDa)*, Mathematics Department "G. Peano", Università degli Studi di Torino.
- December 2016–September 2017. *INdAM Postdoctoral fellow*, Mathematics Department "G. Peano", Università degli Studi di Torino.
- August 2016–November 2016. *Research collaborator*, Futuro in Ricerca 2012 "Differential geometry and Geometric Function Theory", Unità di Ricerca 001 (INdAM).
- November 2014–July 2016. *Postdoctoral fellow*, Algebraic Geometry Sector, Mathematics Department, Università degli Studi di Trento.
- March 2014–October 2014. *Postdoctoral fellow*, Algebra and Algebraic Geometry Sector, Mathematics Department "G. Peano", Università degli Studi di Torino.
- January 2013–December 2013. *Postdoctoral fellow*, Algebra and Algebraic Geometry Sector, Mathematics Department "G. Peano", Università degli Studi di Torino.
- January 2009–December 2011. *Doctoral student*, Mathematics Department "G. Peano", Università degli Studi di Torino.

Academic habilitation

- Italian Habilitation for the position of *Full Professor (Professore di Prima Fascia)*, Geometry and Algebra sector, valid since 2020-11-11 until 2029-11-11.

Publications

Refereed Journal Articles

[KL] *The Gröbner fan of the Hilbert scheme* (with Y. Kambe), Annali di Matematica Pura ed Applicata, 2020.

> **Abstract.** We give a notion of "combinatorial proximity" among strongly stable ideals in a given polynomial ring with a fixed Hilbert polynomial. We show that this notion guarantees "geometric proximity" of the corresponding points in the Hilbert scheme. We define a graph whose vertices correspond to strongly stable ideals and whose edges correspond to pairs of adjacent ideals. Every term order induces an orientation of the edges of the graph. This directed graph describes the behavior of the points of the Hilbert scheme under Gröbner degenerations with respect to the given term order.
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> Then, we introduce a polyhedral fan that we call Gröbner fan of the Hilbert scheme. Each cone of maximal dimension corresponds to a different directed graph induced by a term order. This fan encodes several properties of the Hilbert scheme. We use these tools to present a new proof of the connectedness of the Hilbert scheme. Finally, we improve the technique introduced in the paper "Double-generic initial ideal and Hilbert scheme" by Bertone, Cioffi and Roggero to give a lower bound on the number of irreducible components of the Hilbert scheme.

[AL] *Strongly stable ideals and Hilbert polynomials* (with D. Alberelli), J. Softw. Algebra Geom., **9**:1-9, 2019.

> **Abstract.** The `StronglyStableIdeals.m2` package for *Macaulay2* provides a method to compute all saturated strongly stable ideals in a given polynomial ring with a fixed Hilbert polynomial. A description of the main method and auxiliary tools is given.

[BLS] *The maximum genus problem for locally Cohen-Macaulay space curves* (with V. Beorchia and E. Schlesinger), Milan J. Math., **86**(2):137-155, 2018.

> **Abstract.** Let $P_{\max}(d,s)$ denote the maximum arithmetic genus of a locally Cohen-Macaulay curve of degree d in $\mathbb{P}^3$ that is not contained in a surface of degree $< s$. A bound $P(d,s)$ for $P_{\max}(d,s)$ has been proven by the first author in characteristic zero and then generalized in any characteristic by the third author. In this paper, we construct a large family $\mathcal{C}$ of primitive multiple lines and we conjecture that the generic element of $\mathcal{C}$ has good cohomological properties. From the conjecture it would follow that $P(d,s) = P_{\max}(d,s)$ for $d = s$ and for every $d \geq 2s?1$. With the aid of *Macaulay2* we checked this holds for $s \leq 120$ by verifying our conjecture in the corresponding range.

[BFL] *Truncated modules and linear presentations of vector bundles* (with A. Boralevi and D. Faenzi), Int. Math. Res. Not. IMRN, **17**:5347-5377, 2018.

> **Abstract.** We give a new method to construct linear spaces of matrices of constant rank, based on truncated graded cohomology modules of certain vector bundles as well as on the existence of graded Artinian modules with pure resolutions. Our method allows one to produce several new examples, and provides an alternative point of view on the existing ones.

[FGL] *A computational approach to the ample cone of moduli spaces of curves* (with C. Fontanari and R. Ghiloni), Internat. J. Algebra Comput., **28**(1): 37-51, 2018.

> **Abstract.** We present an alternate proof, much quicker and more straightforward than the original one, of a celebrated Fulton's conjecture on the ample cone of the moduli space of stable rational curves with n marked points in the case $n = 7$.

[LR2] *On the functoriality of marked families* (with M. Roggero), J. Comm. Algebra, **8**(3):367–410, 2016.

> **Abstract.** The application of methods of computational algebra has recently introduced new tools for the study of Hilbert schemes. The key idea is to define flat families of ideals endowed with a scheme structure whose defining equations can be determined by an algorithmic procedure. In order to obtain families consistent with the scheme structure of the Hilbert scheme, this construction has to work on polynomial rings with coefficients in any $\mathbb{Z}$-algebra. A natural family of ideals satisfying these requirements is represented by the so-called Gröbner stratum, that is the family of ideals sharing the same initial ideal with respect to a given term ordering, which in general is a locally closed subscheme of the Hilbert scheme. Considering Borel-fixed ideals as initial ideals, some Gröbner strata turn out to be open subsets of the Hilbert scheme, but it has been showed that, except few cases, they do not cover the Hilbert scheme. For this reason, several authors developed new algorithmic procedures, based on the combinatorial properties of Borel-fixed ideals, that allow to associate to each Borel-fixed ideal a flat family of ideals more general, and so usually larger, than the Gröbner stratum. These families, called marked families, correspond to open subsets of the Hilbert scheme and they cover the Hilbert scheme. In this paper, we give a functorial foundation of the construction of marked families, showing that the algorithmic procedures introduced in previous papers do not depend on the coefficient ring.

[BLMR] *Extensors and the Hilbert scheme* (with J. Brachat, B. Mourrain and M. Roggero), Ann. Sc. Norm. Super. Pisa Cl. Sci. (5), Vol. XVI(1), 2016.

Abstract. The Hilbert scheme $\mathbf{Hilb}^n_{p(t)}$ parametrizes closed subschemes and families of closed subschemes in the projective space $\mathbb{P}^n$ with a fixed Hilbert polynomial $p(t)$. It is classically realized as a closed subscheme of a Grassmannian or a product of Grassmannians. In this paper we consider schemes over a field k of characteristic zero and we present a new proof of the existence of the Hilbert scheme as a subscheme of the Grassmannian $\mathbf{Gr}^{N(r)}_{p(r)}$, where $N(r) = h^0(\mathcal{O}_{\mathbb{P}^n}(r))$. Moreover, we exhibit explicit equations defining it in the Plücker coordinates of the Plücker embedding of $\mathbf{Gr}^{N(r)}_{p(r)}$. Our proof of existence does not need some of the classical tools used in previous proofs, as flattening stratifications and Gotzmann's Persistence Theorem. The degree of our equations is deg $p(t) + 2$, lower than the degree of the equations given by Iarrobino and Kleiman in 1999 and also lower (except for the case of hypersurfaces) than the degree of those proved by Haiman and Sturmfels in 2004 after Bayer's conjecture in 1982. The novelty of our approach mainly relies on the deeper attention to the intrinsic symmetries of the Hilbert scheme and on some results about Grassmannian based on the notion of extensors.

[CLM] *A combinatorial description of finite O-sequences and aCM genera* (with F. Cioffi and M. G. Marinari), J. Symbolic Comput., **73**:104–119, 2016.

Abstract. The goal of this paper is to explicitly detect all the arithmetic genera of arithmetically Cohen-Macaulay projective curves with a given degree d. It is well-known that the arithmetic genus g of a curve C can be easily deduced from the h-vector of the curve; in the case where C is arithmetically Cohen-Macaulay of degree d, g must belong to the range of integers $\{0, \dots, \binom{d-1}{2}\}$. We develop an algorithmic procedure that allows one to avoid constructing most of the possible h-vectors of C. The essential tools are a combinatorial description of the finite O-sequences of multiplicity d, and a sort of continuity result regarding the generation of the genera. The efficiency of our method is supported by computational evidence. As a consequence, we single out the minimal possible Castelnuovo-Mumford regularity of a curve with Cohen-Macaulay postulation and given degree and genus.

[AAGL] *The maximum likelihood degree of Fermat hypersurfaces* (with D. Agostini, D. Alberelli and F. Grande), J. Algebr. Stat., **6**(2):108–132, 2015.

Abstract. We study the critical points of the likelihood function over the Fermat hypersurface. This problem is related to one of the main problems in statistical optimization: maximum likelihood estimation. The number of critical points over a projective variety is a topological invariant of the variety and is called maximum likelihood degree. We provide closed formulas for the maximum likelihood degree of any Fermat curve in the projective plane and of Fermat hypersurfaces of degree 2 in any projective space. Algorithmic methods to compute the ML degree of a generic Fermat hypersurface are developed throughout the paper. Such algorithms heavily exploit the symmetries of the varieties we are considering. A computational comparison of the different methods and a list of the maximum likelihood degrees of several Fermat hypersurfaces are available in the last section.

[CLMR2] *Minimal Castelnuovo-Mumford regularity for a given Hilbert polynomial* (with F. Cioffi, M. G. Marinari and M. Roggero), Exp. Math., **24**(4):424–437, 2015.

Abstract. Let K be an algebraically closed field of null characteristic and $p(z)$ a Hilbert polynomial. We look for the minimal Castelnuovo-Mumford regularity $m_{p(z)}$ of closed subschemes of projective spaces over K with Hilbert polynomial $p(z)$. Experimental evidences led us to consider the idea that $m_{p(z)}$ could be achieved by schemes having a suitable *minimal Hilbert function*. We give a constructive proof of this fact. Moreover, we are able to compute the minimal Castelnuovo-Mumford regularity $m^{\varrho}_{p(z)}$ of schemes with Hilbert polynomial $p(z)$ and given regularity ϱ of the Hilbert function, and also the minimal Castelnuovo-Mumford regularity m_u of schemes with Hilbert function u.

These results find applications in the study of Hilbert schemes. They are obtained by means of *minimal Hilbert functions* and of two new constructive methods which are based on the notion of growth-height-lexicographic Borel set and called *ideal graft* and *extended lifting*.

[HLS] *Smooth curves specialize to extremal curves* (with R. Hartshorne and E. Schlesinger), Math. Ann., 361(1):459–476, 2015.

Abstract. Let $H_{d,g}$ denote the Hilbert scheme of locally Cohen-Macaulay curves of degree d and genus g in projective three space. We show that, given a smooth irreducible curve C of degree d and genus g, there is a rational curve $\{[C_t] : t \in \mathbb{A}^1\}$ in $H_{d,g}$ such that C_t for $t \neq 0$ is projectively equivalent to C, while the special fibre C_0 is an extremal curve. It follows that smooth curves lie in a unique connected component of $H_{d,g}$. We also determine necessary and sufficient conditions for a locally Cohen-Macaulay curve to admit such a specialization to an extremal curve.

[LS] *The Hilbert schemes of locally Cohen–Macaulay curves in* $\mathbb{P}^3$ *may after all be connected* (with E. Schlesinger), Collect. Math., 64(3):363–372, 2013.

Abstract. Progress on the problem whether the Hilbert schemes of locally Cohen-Macaulay curves in $\mathbb{P}^3$ are connected has been hampered by the lack of an answer to a question raised by Robin Hartshorne in (Commun. Algebra 28:6059–6077, 2000) and more recently in (American Institute of Mathematics, Workshop components of Hilbert schemes, problem list, 2010. `http://aimpl.org/hilbertschemes`): does there exist a flat irreducible family of curves whose general member is a union of $d \geqslant 4$ disjoint lines on a smooth quadric surface and whose special member is a locally Cohen-Macaulay curve in a double plane? In this paper we give a positive answer to this question: for every d we construct a family with the required properties, whose special fiber is an extremal curve in the sense by Martin-Deschamps and Perrin (Ann. Sci. E.N.S. 4^e Série 29:757-785, 1996). From this we conclude that every effective divisor in a smooth quadric surface is in the connected component of its Hilbert scheme that contains extremal curves.

[BLR] *A Borel open cover of the Hilbert scheme* (with C. Bertone and M. Roggero), J. Symbolic Comput., 53:119–135, 2013.

Abstract. Let $p(t)$ be an admissible Hilbert polynomial in $\mathbb{P}^n$ of degree d. The Hilbert scheme $\mathcal{H}\mathrm{ilb}^n_{p(t)}$ can be realized as a closed subscheme of a suitable Grassmannian $\mathbb{G}$, hence it could be globally defined by homogeneous equations in the Plücker coordinates of $\mathbb{G}$ and covered by open subsets given by the non-vanishing of a Plücker coordinate, each embedded as a closed subscheme of the affine space $\mathbb{A}^D$, $D = \dim(\mathbb{G})$. However, the number E of Plücker coordinates is so large that effective computations in this setting are practically impossible. In this paper, taking advantage of the symmetries of $\mathcal{H}\mathrm{ilb}^n_{p(t)}$, we exhibit a new open cover, consisting of marked schemes over Borel-fixed ideals, whose number is significantly smaller than E. Exploiting the properties of marked schemes, we prove that these open subsets are defined by equations of degree $\leqslant d+2$ in their natural embedding in $\mathbb{A}^D$. Furthermore we find new embeddings in affine spaces of far lower dimension than D, and characterize those that are still defined by equations of degree $\leqslant d+2$. The proofs are constructive and use a polynomial reduction process, similar to the one for Gröbner bases, but are term order free. In this new setting, we can achieve explicit computations in many non-trivial cases.

[BCLR] *Upgraded methods for the effective computation of marked schemes on a strongly stable ideal* (with C. Bertone, F. Cioffi and M. Roggero), J. Symbolic Comput., 50:263–290, 2013.

Abstract. Let $J \subset S = K[x_0, \ldots, x_n]$ be a monomial strongly stable ideal. The collection $\mathcal{M}\mathrm{f}(J)$ of the homogeneous polynomial ideals I, such that the monomials outside J form a K-vector basis of S/I, is called a J-marked family. It can be endowed with a structure of affine scheme, called a J-marked scheme. For special ideals J, J-marked schemes provide an open cover of the Hilbert scheme $\mathcal{H}\mathrm{ilb}^n_{p(t)}$, where $p(t)$ is the Hilbert polynomial of S/J. Those ideals more suitable to this aim are the m-truncation ideals $\underline{J}_{\geqslant m}$ generated by the monomials of degree $\geqslant m$ in a saturated strongly stable monomial ideal $\underline{J}$. Exploiting a characterization of the ideals in $\mathcal{M}\mathrm{f}(\underline{J}_{\geqslant m})$ in terms of a Buchberger-like criterion, we compute the equations defining the $\underline{J}_{\geqslant m}$-marked scheme by a new reduction relation, called superminimal reduction, and obtain an embedding of $\mathcal{M}\mathrm{f}(\underline{J}_{\geqslant m})$ in an affine space of low dimension. In this setting, explicit computations are achievable in many non-trivial cases. Moreover, for every m, we give a closed embedding $\phi_m : \mathcal{M}\mathrm{f}(\underline{J}_{\geqslant m}) \hookrightarrow \mathcal{M}\mathrm{f}(\underline{J}_{\geqslant m+1})$, characterize those ϕ_m that are isomorphisms in terms of the monomial basis of $\underline{J}$, especially we characterize the minimum integer m_0 such that ϕ_m is an isomorphism for every $m \geqslant m_0$.

[CLMR] *Segments and Hilbert schemes of points* (with F. Cioffi, M. G. Marinari and M. Roggero), Discrete Math., 311(20):2238–2252, 2011.

Abstract. Using results obtained from a study of homogeneous ideals sharing the same initial ideal with respect to some term order, we prove the singularity of the point corresponding to a segment ideal with respect to a degreverse term order (as, for example, the degrevlex order) in the Hilbert scheme of points in $\mathbb{P}^n$. In this context, we look into the properties of several types of "segment" ideals that we define and compare. This study also leads us to focus on the connections between the shape of generators of Borel ideals and the related Hilbert polynomial, thus providing an algorithm for computing all saturated Borel ideals with a given Hilbert polynomial.

[LR] *Rational components of Hilbert schemes* (with M. Roggero), Rend. Semin. Mat. Univ. Padova, 126:11–45, 2011.

Abstract. The Gröbner stratum of a monomial ideal j is an affine variety that parameterizes the family of all ideals having j as initial ideal (with respect to a fixed term ordering). The Gröbner strata can be equipped in a natural way with a structure of homogeneous variety and are in a close connection with Hilbert schemes of subschemes in the projective space $\mathbb{P}^n$. Using properties of the Gröbner strata we

prove some sufficient conditions for the rationality of components of $\mathcal{H}\text{ilb}^n_{p(t)}$. We show for instance that all the smooth, irreducible components in $\mathcal{H}\text{ilb}^n_{p(t)}$ (or in its support) and the Reeves and Stillman component H_{RS} are rational. We also obtain sufficient conditions for isomorphisms between strata corresponding to pairs of ideals defining a same subscheme, that can strongly improve an explicit computation of their equations.

[FLL] *Gaussian curves for visualizing chronological patterns of ceramic finds and residuality* (with A. Ferrarese Lupi), Archaeometry, 55(2):296–311, 2013.

> **Abstract.** This paper describes a particular statistical approach to chronological data from assemblages of archaeological finds (namely pottery) using Gaussian curves: the method enables us to obtain a graphic representation of chronological patterns that avoids an excessive flattening of data, adding an extra dimension to the usual linear temporal concepts. This enables us to arrive at more likely absolute dating and also visualize residuality. The method has been built during the study of a stratigraphic context from the Site of Ancient Ships of Pisa–San Rossore, Italy, and applied to its pottery. Archaeologists may find in the model a useful tool that is easy to apply.

Refereed Proceedings

[Lel] *An efficient implementation of the algorithm computing the Borel-fixed points of a Hilbert scheme*, ISSAC 2012 — Proceedings of the 37th International Symposium on Symbolic and Algebraic Computation, pages 242–248, 2012.

> **Abstract.** Borel-fixed ideals play a key role in the study of Hilbert schemes. Indeed each component and each intersection of components of a Hilbert scheme contains at least one Borel-fixed point, i.e. a point corresponding to a subscheme defined by a Borel-fixed ideal. Moreover Borel-fixed ideals have good combinatorial properties, which make them very interesting in an algorithmic perspective. In this paper, we propose an implementation of the algorithm computing all the saturated Borel-fixed ideals with number of variables and Hilbert polynomial assigned, introduced from a theoretical point of view in the paper "Segment ideals and Hilbert schemes of points", Discrete Mathematics 311 (2011).

Preprints

[BLF2] *A construction of equivariant bundles on the space of symmetric forms* (with A. Boralevi and D. Faenzi), `arXiv:1804.06211`.

> **Abstract.** We construct stable vector bundles on the space of symmetric forms of degree d in $n+1$ variables which are equivariant for the action of $\mathrm{SL}_{n+1}(\mathbb{C})$, and admit an equivariant free resolution of length 2. For $n = 1$, we obtain new examples of stable vector bundles of rank $d-1$ on $\mathbb{P}^d$, which are moreover equivariant for $\mathrm{SL}_2(\mathbb{C})$. The presentation matrix of these bundles attains Westwick's upper bound for the dimension of vector spaces of matrices of constant rank and fixed size.

Manuscripts in preparation

[GL] *The turning principle and its pervasive effect in the theory of slice functions* (with R. Ghiloni).

[L1] *On the tangent space to the Hilbert scheme*.

[L2] `MarkedSchemes.m2`. *A Macaulay2 package for marked families and Gröbner strata*.

[BCL] *Computing marked families via syzygy forecasting* (with C. Bertone and F. Cioffi).

Seminars & Presentations

Conference Invited Talks

- Workshop "Quaternioni sul Conero", *Approximation of slice functions*, Ancona (Italy), September 7-8, 2017.
- Workshop "Combinatorial Moduli Space", event of the thematic program Combinatorial Algebraic Geometry, *Local and global equations of the Hilbert scheme*, Fields Institute, Toronto (Canada), December 5-9, 2016.
- Conference "Recent progress in real and complex geometry – X", *Towards Fulton's conjecture on the ample cone of the moduli space of rational curves with marked points*, Levico Terme (TN), Italy, October 8-11, 2016.

- Conference "Classification of Projective Varieties", *Symmetry and equations of the Hilbert scheme*, Levico Terme (TN), Italy, September 2015.
- Mini-Symposium "Maximum Likelihood Degrees and Critical Points" (subevent of "SIAM Conference on Applied Algebraic Geometry"), *The maximum likelihood degree of Fermat hypersurfaces*, Daejeon (South Korea), August 2015.
- Conference "Giornate di Geometria Algebrica ed Argomenti Correlati XI", *Sul problema della connessione di schemi di Hilbert vari*, Pisa (Italy), May 2012.
- Workshop "G.T.M. Seminar", *Borel-fixed ideals and Hilbert schemes*, Genova (Italy), February 2011.
- Workshop "A St. John geometry day in Turin", *Deformations of Borel-fixed ideals and rational curves on the Hilbert scheme*, Torino (Italy), June 2010.

Invited seminars

- Università degli Studi di Napoli, *Construction of spaces of matrices of constant rank*, June 2016.
- Università degli Studi di Torino, *Construction of spaces of matrices of constant rank*, March 2016.
- Università degli Studi di Trento, *Development and application of methods of computational algebra fir the study of algebraic varieties*, October 2015.
- Technische Universiteit Eindhoven, *The maximum likelihood degree of Fermat hypersurfaces*, May 2015.
- Università degli Studi di Trento, *Il teorema di connessione dello schema di Hilbert*, November 2014.
- Politecnico di Milano, *The maximum likelihood degree of Fermat hypersurfaces*, May 2014.
- Università degli Studi di Napoli "Federico II", *Smooth curves specialize to extremal curves*, December 2012.
- Università degli Studi di Torino, *Smooth curves specialize to extremal curves*, November 2012.
- Università degli Studi di Trieste, *Equations for the Hilbert scheme*, October 2012.
- Politecnico di Milano, *Equations for the Hilbert scheme*, January 2012.
- Politecnico di Torino, *Rationality of components of the Hilbert scheme*, March 2009.

Contributed Talks *(Accepted talk proposals)*

- Congress "XX Congresso dell'Unione Matematica Italiana", *Locally Cohen-Macaulay curves in* $\mathbb{P}^3$ *supported on a line*, Siena (Italy), September 2015.
- Conference "MEGA 2013", Frankfurt am Main, *Explicit construction of degenerations of space curves to extremal curves*, June 2013.
- Conference "ISSAC 2012 – 37th International Symposium on Symbolic and Algebraic Computation", Grenoble, *An efficient implementation of the algorithm computing the Borel-fixed points of a Hilbert scheme*, July 2012.

Posters

- "3rd Carlo Alberto Stochastics Workshop", Torino, *The maximum likelihood degree of Fermat hypersurfaces* (with F. Grande), January 2014.
- Workshop "Syzygies in Berlin", Berlin, *Groebner degenerations of space curves to extremal curves*, May 2013.

Other presentations

- Politecnico di Milano, *Spaces of constant rank matrices*, May 2018.
- Accademia delle Scienze di Torino, Premio Fubini ceremony, *Points of view on conics*, October 2016.
- Università di Torino, Mathematics PhD Seminars, *Plane conics. Enumerative problems and the Hilbert scheme*, October 2014.
- Università di Torino, Working seminar "Moduli spaces of curves", 3 lectures, November-December 2013.
- Università di Torino, Mathematics PhD Seminars, *Polynomial algebra and Sudoku*, October 2013.
- Workshop "Un pomeriggio di statistica algebrica", Torino, *Solving polynomial equations by homotopic continuation*, July 2013.

Teaching & Supervising Experience

Supervisor

- Mentor of Yuta Kambe during his period as PhD Visiting Student at Mathematics Department of Politecnico di Milano (since 2018/09/21 until 2018/12/09 and since 2019/05/09 until 2019/07/25).
- Advisor of the Master thesis (Mathematical Engineering) of Stefano Spaziani. Politecnico di Milano, discussion expected for March 2021.
- Supervisor of the reading course (bachelor thesis, Mathematical Engineering) of Gilda Matteucci. Title: "Learning the parts of objects by non-negative matrix factorization". Politecnico di Milano, September 2019.
- Supervisor of the reading course (bachelor thesis, Mathematical Engineering) of Daniele Mulè. Titolo: "Eckart-Young theorem and image compression". Politecnico di Milano, June 2019.
- Co-advisor of the Master thesis (Mathematics) of Davide Trotta. Title: "Stability of coherent sheaves on curves". Università degli Studi di Trento, October 2016.
- Supervisor of the Bachelor thesis (Mathematics) of Stefano Fessler. Title: "Max-Plus Algebra". Università degli Studi di Trento, September 2016.

Graduate level

- Spring 2021, Politecnico di Milano, PhD School Mathematical Models and Methods in Engineering, *Topological Data Analysis*.

Undergraduate level

- Spring 2021, Politecnico di Milano, Scuola di Ingegneria Industriale e dell'informazione, *Analisi e Geometria 2 (Calculus and Geometry 2)*.
- Fall 2020, Politecnico di Milano, Scuola di Ingegneria Industriale e dell'informazione, *Topologia Algebrica Computazionale (Computational algebraic topology)*.
- Spring 2020, Politecnico di Milano, Scuola di Ingegneria Industriale e dell'informazione, *Algebra Lineare e Geometria (Linear Algebra and Geometry)*.
- Spring 2020, Politecnico di Milano, Scuola di Ingegneria Industriale e dell'informazione, *Analisi e Geometria 2 (Calculus and Geometry 2)*.
- Spring 2019, Politecnico di Milano, Scuola di Ingegneria Industriale e dell'informazione, *Analisi e Geometria 2 (Calculus and Geometry 2)*.
- Fall 2018, Politecnico di Milano, Scuola di Ingegneria Civile, Ambientale e Territoriale, *Geometria (Geometry)*.
- Spring 2018, Politecnico di Milano, Scuola di Ingegneria Industriale e dell'informazione, *Analisi e Geometria 2 (Calculus and Geometry 2)*.

- Fall 2017, Università degli Studi di Torino, Corso di Laurea in Scienze Naturali, *Istituzioni di matematiche e metodi statistici*.
- Fall 2014, Università degli Studi di Trieste, *Minicorso introduttivo per l'utilizzo del software Macaulay2*.
- Spring 2014, Università degli Studi di Torino, *Recupero di Geometria UNO*.
- Fall 2012, Università degli Studi di Trieste, *Minicorso introduttivo per l'utilizzo del software Macaulay2*.
- Fall 2010, Università degli Studi di Torino, Facoltà di Agraria, *Corso di azzeramento in matematica*.

Teaching assistant

- Fall 2019, Politecnico di Milano, Scuola di Ingegneria Industriale e dell'informazione, *Geometria differenziale discreta (discrete differential geometry)*.
- Fall 2016, Università degli Studi di Trento, *Geometria B* (first part).
- Fall 2016, Università degli Studi di Trento, *Geometria A* (first part).
- Spring 2016, Università degli Studi di Trento, *Geometria III*.
- Fall 2015, Università degli Studi di Trento, *Geometria A* (first part).
- Spring 2015, Università degli Studi di Trento, *Geometria III*.
- Fall 2012, Università degli Studi di Torino, *Precorso di matematica*.
- Spring 2012, Politecnico di Torino, *Geometry*, (english course).
- Spring 2012, Politecnico di Torino, *Geometria*.
- Spring 2011, Politecnico di Torino, *Geometria*.
- Spring 2009, Politecnico di Torino, *Geometria*.

Tutor

- Fall 2007, Politecnico di Torino, *Geometria 1*.

Courses for high school students

- Campus MFS, *An introduction to probability and algebra through evolutionary gamesi* (con R. Sirovich), December 16-18, 2016, Bardonecchia (Torino).
- La bottega del Matematico, April 26-29, 2016, Salorno (BZ).
- Campus MFS, *An introduction to probability and algebra through evolutionary games* (with R. Sirovich), December 18-20, 2015, Bardonecchia (Torino).
- Campus MFS, *Observing, formalizing and "solving"…an introduction to the role of the mathematician* (with R. Sirovich), April 10-12, 2015, Bard (Aosta).
- Campus MFS, *An introduction to probability and graph theory through evolutionary games* (with R. Sirovich), January 9-11, 2015, Bardonecchia (Torino).
- Campus MFS, *An introduction to probability and graph theory through evolutionary games* (with R. Sirovich), December 19-21, 2014, 2015, Bardonecchia (Torino).

Organizational experience

- Organizer of the online seminar *Monthly seminar on (Hyper)Complex analysis and geometry*, Politecnico di Milano.
- Organizer of the online workshop *Recent progress in HyperComplex analysis and geometry*, Milano, September 10-11, 2020.
- Organizer of the workshop *A two-day journey in Computational Algebra and Algebraic Geometry*, Torino, September 27-28, 2018.
- Organizer of the workshop *Genova-Torino-Milano Seminar: some topics in Commutative Algebra and Algebraic Geometry*, Milano, July 17-18, 2018.
- Organizer of the workshop "Un pomeriggio di statistica algebrica" (An afternoon on algebraic statistics), Mathematics Department, Università degli Studi di Torino, July 10, 2013.
- Co-organizer of the working seminar "Moduli spaces of curves", Università degli Studi e Politecnico di Torino, September 2013-May 2014.

Scholarships, Grants, Service

- *Ph.D. scholarship* from Università di Torino, January 2009–December 2011.
- *Bachelor degree fellowship* from Istituto Nazionale di Alta Matematica "F. Severi" INDAM, 2003-2006.
- Reviewer for Zentralblatt Math and Mathematical Reviews.
- Referee for Journal of Algebraic Statistics, Applicable Algebra in Engineering, Communication and Computing, Journal of Symbolic Computation, Journal of Commutative Algebra, Forum Mathematicum.

Participation in research groups

- *Prin 2008*. Part of the research project PRIN 2008 titled "Geometria delle Varietà Algebriche", group of Torino.
- *GNSAGA–INdAM*. Member of *Gruppo Nazionale per le Strutture Algebriche, Geometriche e le loro Applicazioni* since 2009.
- *Local research groups 2013-2014*. Mathematics Department "G. Peano", Università degli Studi di Torino.
- *Prin 2010-2011*. Part of the research project PRIN 2010-2011 titled "Geometria delle Varietà Algebriche", group of Trento.
- *UMI*. Member of *Unione Matematica Italiana* since 2014.
- *Firb 2012*. External collaborator of the project Futuro in Ricerca 2012 "Differential geometry and geometric function theory".
- *Local research groups 2016-2017*. Mathematics Department "G. Peano", Università degli Studi di Torino.
- Part of the research project *Hypercomplex function theory and applications*, INdAM.

Awards

- Guido Fubini Prize 2016 for the research in algebra.

Personal Information

- *Citizenship:* Italian (Born: Pinerolo, Italy).
- *Date of birth:* February 11th, 1984.
- *Languages:* Italian (native speaker), English (fluent), French (fluent - DELF certificate).
- *Computer skills:* Excellent knowledge of Apple Mac OS X, Linux OS. Excellent knowledge and extensive experience of programming with C, C++, Java, XHTML. Excellent knowledge of mathematical software: LaTeX, Macaulay2, Bertini, Singular, R, Maple, Matlab.